These estimates are very approximate since we have used the data of Ref. 4 for a liquid layer and the data of Ref. 5 for Teflon at room temperature. But they should serve as rough guides. In the aforementioned, we have used the definitions

$$\tau \equiv \mu\kappa; \quad \kappa \equiv \tfrac{2}{5}(\gamma M^2/2G)c_f$$

Conclusions

In previous work on related problems[2] it has been found that the "practical" stability boundary is generally the strong instability found at coalescence, in the present example as $\lambda \to \frac{1}{2}$. Moreover the finite dimensions of the problem and the precise nature of the material elasticity and dissipation have been found to be important at supersonic conditions[6] though less important at subsonic Mach numbers. This suggests that the present model may need certain refinements. For example, one or more of the finite dimensions of the solid may be important or, as suggested by Nachtsheim,[7] the inner solid is really a thin liquid film which generates its own characteristic scale length.

Also we may take the opportunity to evaluate approximations to the high-speed gas flow which have been used by some investigators.[7] These are:

Quasi-steady theory $(\omega/U\alpha \ll 1)$

$$p = e^{\pm i\alpha(M^2-1)^{1/2}y} \tag{4}$$

$$\lambda X^2/[i\varepsilon X + 1] = (M^2-1)^{1/2} \tag{8$_{qs}$}$$

or

Piston Theory $M(1+\omega/U\alpha) \gg 1$

$$p = e^{\pm i\alpha(M+\omega/U\alpha)} \tag{4}$$

$$\lambda X^2/(i\varepsilon X + 1) = X \tag{8$_{pt}$}$$

From Eq. $(8)_{qs}$, solving for X, one concludes the model is always stable. From Eq. $(8)_{pt}$, one obtains

$$X = (\lambda + i\varepsilon)/(\lambda^2+\varepsilon^2) \simeq 1/\lambda + i(\varepsilon/\lambda^2) \quad \text{for} \quad \varepsilon \ll \lambda$$

This latter result is a reasonable approximation to the stable pole as shown in Fig. 2. However, neither of these approximations will give any useful information concerning the unstable pole and, therefore, must be considered inadequate for the present model. Finally we note that Gold, Probstein, and Scullen[5] have recently treated the shear flow velocity profile flow and also considered alternative surface models. Additional such studies would appear warranted particularly with regard to characterizing the surface more accurately. Shear flow effects have also been treated recently in the context of another though similar problem.[8]

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Study of 8–15 km/sec Shock Waves Using Conventional Shock Tubes

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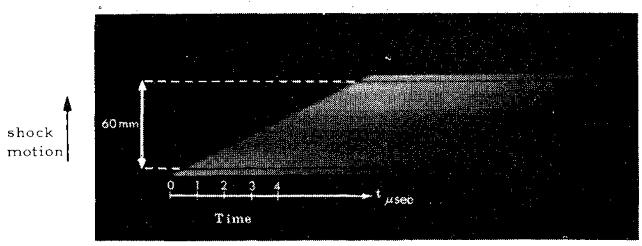
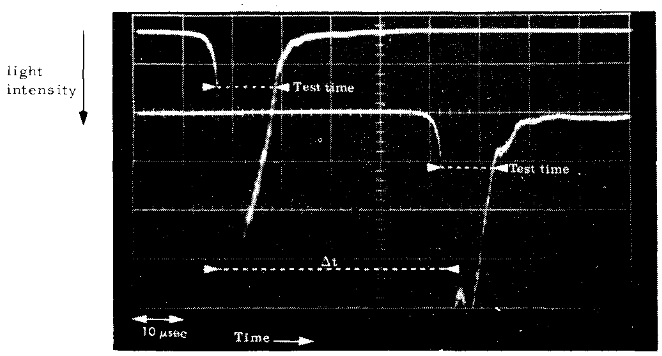
It has been demonstrated[1] that it is possible to increase the shock wave velocity, for a given over-all pressure ratio P_{40}, by dividing this ratio among one or several intermediate chambers. We have studied and operated a shock tube with one so-called middle pressure chamber. The velocity gain g of that tube, compared to the one-diaphragm apparatus, becomes noticeable when P_{40} exceeds 10^5 and when the pressure P_1 of the light gas (H_2 or He) in the intermediate chamber has an optimum value. This gain is approximately 40% when $P_{40} = 10^7$.

The main characteristics of our double diaphragm shock tube are: the combustion of an O_2—H_2—He homogeneous mixture (molar proportions 1-2-7) takes place in the high-pressure chamber, HP (length $L = 0.5$ m; diam $\phi_{\rm int} = 150$ mm), and enables conditions there to reach a few kilobars and 2500°K within a few milliseconds. The middle pressure chamber, MP ($L = 2$ m; $\phi_{\rm int} = 100$ mm), is filled up with H_2. The low-pressure chamber, BP ($L = 12$ m; $\phi_{\rm int} = 100$ mm), is made of stainless steel. The diaphragm D_1 which separates HP and MP chambers is made of stainless steel and scribed on two perpendicular diameters. The diaphragm D_2 between MP and BP chambers is made of thin mylar. One assumes that it opens instantaneously and does not disturb the flow. Some experiments with argon under 0.1 torr show that the shock velocity exceeds 14 km/sec.



a) Continuous writing streak camera 7 m from second diaphragm— $U = 9.4$ mm/μsec



b) Photomultiplier oscillograms: PM are located 9.8 and 10.2 m, respectively, from second diaphragm $U = 400/\Delta t = 9.0$ mm/μsec, test time $\simeq 10\mu$sec

Fig. 1 Shock wave velocity and test time for initial argon pressure $P_0 = 1$ torr.

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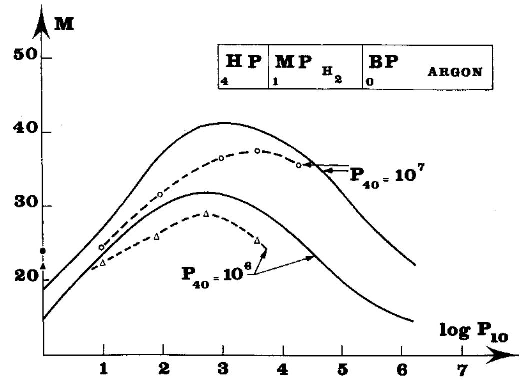

Fig. 2 Mach number vs logarithm of the ratio of intermediate chamber pressure to test chamber pressure for two values of the ratio of driver chamber pressure to test chamber pressure, ——— theoretical curve, ----- experimental curve, ○ and △ experimental points. ● ▲ experimental points without an intermediate chamber.

The experiments here reported deal with argon ($P_0 = 1$ torr) undergoing a shock of $M \simeq 30$. A continuous writing streak camera enables measurement with 1% of the velocity of the wave 7 m away from D_2 diaphragm (Fig. 1a). Two photomultipliers whose distances to D_2 are 9.8 m and 10.2 m, respectively, give the velocity and the test time (Fig. 1b).

The existence of an optimum P_1 pressure has been checked. Figure 2 shows that the optimum value of P_1 is obtained for $P_{10} = 10^3$ when $P_{40} = 10^6$. From experiments at other values of P_{40}, it may be deduced that P_{10} optimum $\simeq P_{40}^{1/2}$.

Measurement of the attenuation of the shock wave velocity is performed through a 35 GHz microwave system. The microwave is reflected by the shock front along the axis of the tube. Because of the precursor phenomena which travels ahead of a strong shock wave, this Doppler effect method gives the correct value of the velocity when the microwave frequency is high enough for the microwave to be reflected near the pressure front. Figure 3 gives the attenuation of the velocity as a function of the distance to D_2 diaphragm for different values of P_0.

On Fig. 3 are shown also the results from some high-performance shock tubes. Arc driven shock tubes[2] give very good velocity and test time. Yet we must observe that helium driver gas is very hot ($T \geqq 2 \times 10^4$ °K). In radiative transfer studies driver gas radiation would have to be taken into account. Highest performances reached until now, have been achieved by explosively driven shock tubes.[3] In these tubes, shock heated gas is quickly disturbed by the expansion wave and so test time is very short. Free-piston double-diaphragm shock tubes[4] produce shock velocities which exceed 17 km/sec. But when the test gas

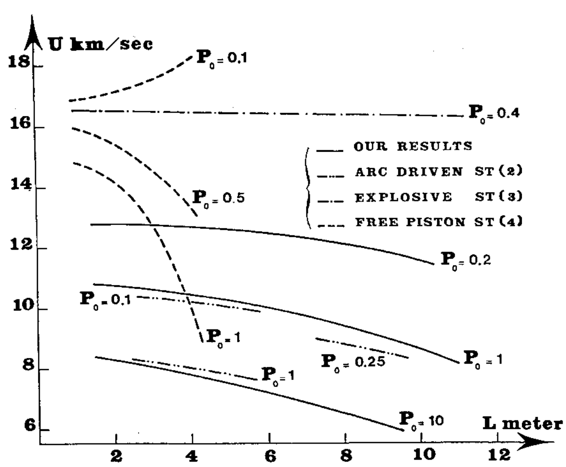

Fig. 3 Attenuation of shock wave velocity along the tube for some high performance shock tubes. P_0 is the initial test gas pressure in torr.

pressure is 1 or 0.5 torr, wave attenuation is very high; when initial test gas pressure is 0.1 torr, shock speed increases along the shock tube. In these circumstances the flow is not steady and radiative transfer problems are very severe. In our shock tube, wave attenuation is weak (200 m/sec per meter at 0.2 torr initial test gas pressure; 250 m/sec per meter at 1 torr initial test gas pressure) and a test-time of 10 μsec is available 10 m distant from D_2 diaphragm.

Use of an intermediate chamber and very high pressure in the driver increase combustion shock tube performance as high as that of powerful arc driven shock tubes.

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Regularization of Grand Tour Trajectories

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Introduction

THE computation of precision Grand Tour interplanetary trajectories is very sensitive to errors introduced by the method of solution. To obtain accurate solutions of trajectories with multiple, consecutive, planetary close approaches, numerical integration of unregularized equations of motion often requires a large number of integration steps and excessive calculation times. To reduce the number of integration steps during each close approach, KS regularization, introduced by Kustaanheimo and Stiefel,[1] is employed. The regularization eliminates the singularities due to the planetary and solar attractions in the equations of motion of a space vehicle.

The simultaneous removal of all mathematical and numerical singularities in the equations of motion of a space vehicle, with both independent and dependent variable transformations, has not yet been accomplished. The KS method removes only one of the singularities. Since the space vehicle encounters the singularities only consecutively and not simultaneously, it appears sufficient to remove the singularities consecutively, one at a time, as the singularities are approached. A regularizing algorithm developed by Szebehely and Peters[2] is employed for this purpose. The algorithm applies the Levi-Civita method of regularization[3] to remove the singularity between the closest

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